PACIFIC JOURNAL OF MATHEMATICS
Vol. 166, No. 1, 1994

ORDER OF THE IDENTITY OF THE STABLE SUMMANDS OF $\Omega^{2k}S^{2n+1}$

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We obtain upper bounds for the stable suspension orders of the summands in Snaith's stable decomposition of $\Omega^{2k}S^{2n+1}$, for $2k < 2n+1$, and localized at a prime p. These finite, torsion complexes also occur as subquotients of May's filtration of $\Omega^{2k}S^{2n+1}$, and as Thom spaces of canonical vector bundles. The results are obtained by induction, using results of Toda on the stable orders of stunted real projective spaces (for $p = 2$) and certain cofibrations of Mahowald to start the induction and proceding by stabilizing factorizations of power maps on $\Omega^{2k}S^{2n+1}$ due to James and Selick for $p = 2$ and Cohen-Moore-Neisendorfer for $p > 2$.

Introduction. For suitable topological spaces X and Y let $[X, Y]$ be the set of homotopy classes of pointed maps from X to Y. It is well known that $[X, Y]$ is a group if the domain is a suspension space and an abelian group if it is a double suspension. In particular, $[\Sigma X, \Sigma X]$ is a group and we say that X has suspension order k if the identity map of ΣX has order k in this group and that X has stable order k if the identity map of $\Sigma^m X$ has order k for m sufficiently large. It is not hard to show that a finite CW-complex whose reduced integral homology is all torsion has finite suspension order.

In this paper we compute upper bounds for the stable orders of a particular class of finite complexes. A theorem of Snaith [**Sn**] gives that $\Omega^n\Sigma^n X$ is stably equivalent to a wedge $\bigvee_{j\geqslant 1} D_j(\Omega^n\Sigma^n X)$, where $D_j(\Omega^n\Sigma^n X)$ is the j-adic construction on X, defined by $D_j(\Omega^n\Sigma^n X) = \mathcal{C}_n(j)\times_{\Sigma_j} X^{[j]}/\mathcal{C}_n(j)\times_{\Sigma_j} *$, where $\mathcal{C}_n(j)$ is the space of "j little n-cubes" in $\mathbf{R}^n$. The spaces $D_j(\Omega^n\Sigma^n X)$ also occur