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PAIRED CALIBRATIONS APPLIED TO SOAP FILMS, IMMISCIBLE FLUIDS, AND SURFACES OR NETWORKS MINIMIZING OTHER NORMS

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In this paper we introduce a new method for proving area-minimization which we call "paired calibrations." We begin with the simplest application, the cone over the tetrahedron, which appears in soap films. We then discuss immiscible fluid interfaces, crystal surfaces, and one-dimensional networks minimizing other norms.

1. Introduction In her classification of soap-film singularities [**T1**], Jean Taylor proved only by the process of elimination that the cone over the edges of the regular tetrahedron minimizes area among surfaces separating the four faces. We give a direct proof which applies to regular simplices in all dimensions. See Figure 1.0.1.

Configurations of several immiscible fluids try to minimize an energy proportional to interfacial surface area, but the constant of proportionality varies for each pair of fluids. Chapter 2 proves that certain cones minimize such weighted areas.

The surface energy of a crystal depends on direction, as given by a norm Φ on unit normals. Chapter 3 proves certain cones Φ-minimizing, such as a cone over a triangular prism. The hypotheses involve basic geometric questions, such as the number of possible cardinalities of equilateral sets (i.e., sets of pairwise equidistant points) for a norm on R^n.

We also consider 1-dimensional Φ-minimizing networks for differentiable norms Φ. It is well-known that length-minimizing networks meet in threes at $120°$ angles. Chapter 4 classifies the singularities in Φ-minimizing networks in R^n and establishes $n+1$ as the sharp bound on the number of segments that can meet at a point.