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ON THE COMPACTNESS OF A CLASS OF RIEMANNIAN MANIFOLDS

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A class of Riemannian manifolds is studied in this paper. The main conditions are 1) the injectivity is bounded away from 0; 2) a norm of the Riemannian curvature is bounded; 3) volume is bounded above; 4) the Ricci curvature is bounded above by a constant divided by square of the distance from a point. Note the last condition is scaling invariant. It is shown that there exists a sequence of such manifolds whose metric converges to a continuous metric on a manifold.

Introduction. Let $\mathcal{L} = \mathcal{L}(H, K, V, n, i_0)$ be the set of n-dimensional Riemannian manifolds (M, g), *s.t.*,

(0.1) M is diffeomorphic to (B_2, g_0), the standard Euclidean ball of radius 2, center $= 0$;
(0.2) (M, g) has C^∞ curvature tensor in M;
(0.3) for any $x \in M$, the Ricci curvature at x $|Ric(g)(x)| \leq Hr^{-2}$, where $r = dist(x, 0)$;
(0.4) the injectivity of $(M, g) \geq i_0 > 0$;
(0.5) $\int_M |Rm(g)|^{\frac{n}{2}} dg < K$;
(0.6) volume of $(M, g) \leq V$.

In the case when the condition (0.3) is replaced by $|Ric(g)| \leq H$, and (0.6) is replaced by a diameter bound, a compactness property is proved by the first author in a more general setting. The purpose of this paper is to extend some of his results to the present situation where the bound om Ricci curvature of (M, g) blows up like r^{-2} at a point. As an application, we will discuss the compactness of orbifolds with a finite number of singularities.