PACIFIC JOURNAL OF MATHEMATICS
Vol. 166, No. 1, 1994

EIGENVALUE BOUNDS AND GIRTHS OF GRAPHS OF FINITE, UPPER HALF-PLANES

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For each odd prime power $q = p^r$ we will investigate $q-2$ different Cayley graphs called finite, upper half planes over F_q. We define a finite, upper half-plane by

$$H_q = \{x + y\sqrt{d} | x, y \in F_q, y \neq 0\}$$

where F_q is the finite field with q elements and d is not a square in F_q. We define a distance k between points z and w $\in H_q$ by

$$k(z, w) = \frac{N(z-w)}{(Imz)(Imw)}$$

where $Nz = z\bar{z}$ and $\bar{z} = x - y\sqrt{d}$ and Re $z = x$ and Im$z = y$. We define a graph, $X_q(d, a)$, by letting the elements of H_q be the vertices of the graph and defining an edge between z and w where $k(z, w) = a$ for a fixed $a \in F_q - \{0\}$. We consider the origin to be the point $\sqrt{d}$. We call $H_q(d, a)$, the finite upper half-plane depending on a fixed a and d. We first concern ourselves with whether the eigenvalues, λ, of the adjacency matrices of the graphs satisfy the Ramanujan bound $|\lambda| \leq \sqrt{q}$. Since the graphs are of degree $q + 1$, the paper shows a method to use the representations for the additive and multiplicative groups of each F_q to find the smaller associated isospectral matrices. We then find the eigenvalues of the isospectral matrices. A computer program has verified the Ramanujan bound for most of the graphs up to the prime power 3^5. We next concern ourselves with the girth of the graphs. This paper shows that the girths are either 3 or 4 and shows that the girth is 3 if $a = 2d$ and $q \equiv 3(mod4)$ or if a and $a - 3d$ are squares in F_q. The girth is 4 if $a = 2d$ and $q \equiv 1(mod4)$.

Nicholas M Katz [7] has proven that the eigenvalues of the graphs do satisfy the Ramanujan bound in the paper "Estimates for Soto-Andrade Sums-1". Graphs whose eigenvalues satisfy the Ramanujan