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APPROXIMATELY INNER AUTOMORPHISMS ON INCLUSIONS OF TYPE III_λ-FACTORS

CARL WINSLØW

For arbitrary inclusions of factors with finite index, we define a "fundamental homomorphism" which is a generalization of both the Connes-Takesaki fundamental homomorphism for properly infinite (single) factors and Loi's construction for inclusions of type II_1-factors.

It is shown that for nice inclusions of type III_λ-factors ($0 < \lambda < 1$), the kernel of the fundamental homomorphism coincides with the set of approximately inner automorphisms on the inclusion. To prove this, we first give a characterization of approximate innerness on type III_λ-inclusions in terms of Loi's and Connes-Takesaki's invariants.

1. Introduction. The importance of studying automorphisms on von Neumann algebras was highlighted through Connes' classification theory for type III-factors. Recently it has been suggested to generalize Connes' automorphism approach to subfactor theory (see e.g. [**Ka1**],[**L2**]).

In Connes' theory, an important class of automorphisms on a von Neumann algebra M is $\overline{\mathrm{Int}}(M)$, the closure — in u-topology, as usual — of $\mathrm{Int}(M)$ in $\mathrm{Aut}(M)$; members of this set are called *approximately inner*. Assume M is a hyperfinite factor. If M is of type I or II_1, then $\overline{\mathrm{Int}}(M) = \mathrm{Aut}(M)$, but if M is of type II_∞ or III, one has $\overline{\mathrm{Int}}(M) = \mathrm{Ker}(\mathrm{mod})$, where mod is the fundamental homomorphism of Connes and Takesaki (see [**CT**, IV.1]). For type III-factors, this was announced by Connes in 1975, and the first published proof was given recently in [**KST**]. The result had prominent applications long before a proof appeared, cf. [**KST**, §0]. As another recent development along these lines, we mention [**HS**], which will be crucial here.