PACIFIC JOURNAL OF MATHEMATICS
Vol. 166, No. 2, 1994

ON DIVISORS OF SUMS OF INTEGERS V

A. SÁRKÖZY AND C. L. STEWART

Dedicated to Professor P. Erdős on the occasion of his eightieth birthday.

Let N be a positive integer and let A and B be subsets of $\{1, \ldots, N\}$. In this article we shall estimate both the maximum and the average of $\omega(a+b)$, the number of distinct prime factors of $a+b$, where a and b are from A and B respectively.

1. Introduction. For any set X let $|X|$ denote its cardinality and for any integer n larger than one let $\omega(n)$ denote the number of distinct prime factors of n. Let I be an integer larger than one and let ϵ be a positive real number. Let $2 = p_1, p_2, \ldots$ be the sequence of prime numbers in increasing order and let m be that positive integer for which $p_1 \cdots p_m \leq N \leq p_1 \cdots p_{m+1}$. In [**3**], Erdős, Pomerance, Sárközy and Stewart proved that there exist positive numbers C_0 and C_1 which are effectively computable in terms of ϵ, such that if N exceeds C_0 and A and B are subsets of $\{1, \ldots, N\}$ with $(|A||B|)^{1/2} > \epsilon N$ then there exist integers a from A and b from B for which

$$\omega(a+b) > m - C_1\sqrt{m}.$$

They also showed that there is a positive real number ϵ, with $\epsilon < 1$, and an effectively computable positive number C_2 such that for each positive integer N there is a subset A of $\{1, \ldots, N\}$ with $|A| \geq \epsilon N$ for which

$$\max_{a,a' \in A} \omega(a+a') < m - \frac{C_2\sqrt{m}}{\log m}.$$

Notice by the prime number theorem that

$$m = (1+o(1))(\log N)/(\log\log N).$$