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PERIODICITY, GENERA AND ALEXANDER POLYNOMIALS OF KNOTS

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For knots in S^3 criteria for periodicity are obtained in terms of the homology groups of cyclic branched covers of S^3 and the Alexander polynomial. Also the relationship between the genus of a periodic knot and the Alexander polynomial is studied. As an application it is shown that no eleven crossing knot has a period greater than 5.

1. Introduction. A knot K in S^3 is said to be *periodic* if there exists an integer $q > 1$ and an orientation preserving diffeomorphism $f : S^3 \to S^3$ such that $f(K) = K$, $\mathrm{order}(f) = q$, and the fixed point set of f is a circle disjoint from K. Any such q is called a *period* of K, and any such f, a corresponding *periodic transformation.*

A natural question is how to determine whether a knot is periodic with a given period. R. Fox [**8**] conjectured in 1962 that a nontrivial knot has only finitely many periods. This was first proved by E. Flapan [**7**] in 1983 and an explicit bound for the possible periods of a knot, in terms of its genus, was given by A. Edmonds [**6**] in 1984.

In 1971, K. Murasugi [**13**] showed that the Alexander polynomial of a period q knot has to satisfy certain conditions. Several other techniques to determine the possible periods for a given knot have been developed since then. The more recent results include criteria involving other polynomial invariants (see [**14**, **16**, **18**, **19**, **20**, **21**]) and hyperbolic structures on knot complements [**1**]. The efficacy of the Murasugi conditions lies in the simplicity of these conditions, the computability of the Alexander polynomial, and the fact that an Alexander polynomial occurs as the polynomial of an infinite collection of knots, thereby making the results applicable to a large class.