PACIFIC JOURNAL OF MATHEMATICS
Vol. 166, No. 2, 1994

IMMERSIONS UP TO JOINT-BORDISM

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A necessary and sufficient condition for a map to be joint-bordant to an immersion is given in terms of Stiefel-Whitney numbers.

1. Introduction. This note is devoted to a study of immersions of manifolds into manifolds up to joint-bordism. We will work throughout in the category of smooth manifolds and smooth maps.

A map of dimension (n, k) is a map of a closed n-manifold into a closed $(n+k)$-manifold. Two maps $f_0 : M_0 \to N_0$ and $f_1 : M_1 \to N_1$ of dimension (n, k) are said to be *joint-bordant* if there is a map $F : V \to W$ extending $f_0 \cup f_1$ where V and W are compact manifolds with $\partial V = M_0 \cup M_1$ and $\partial W = N_0 \cup N_1$. Joint-bordism classes of maps of dimension (n, k) form an abelian group under the disjoint union which we denote by $M(n, k)$. It is well known that Stiefel-Whitney numbers form a complete system of invariants for the joint-bordism theory [**9**]. So one may hope to characterize maps joint-bordant to immersions or embeddings in terms of these numbers whenever $k > 0$. For the case of embeddings this has already been settled by Brown [**3**]; his proof is based on a construction suggested by Stong. In this note, using the *model construction* of Koschorke [**6**], we shall give such a criterion for maps joint-bordant to immersions in the "metastable" range $n \leq 2k$.

Our method of proof can also be applied to study immersions up to various oriented joint-bordism relations. These are naturally defined for the following restricted classes of maps (see Stong [**9**]):

C_1: maps with oriented source manifolds;
C_2: maps with oriented target manifolds;
C_3: maps with oriented stable normal bundles;
C_4: maps with oriented source and target manifolds.