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ON FLATNESS OF THE COXETER GRAPH E_8

MASAKI IZUMI

We will show flatness of Ocneanu's connections on Coxeter graph E_8. This completes classification of subfactors of the type II_1 AFD factor with indices less than 4, which has been stated by A. Ocneanu.

1. Introduction and main results. Since his celebrated work [**J**], V. Jones theory of index is one of the central topics of the theory of operator algebras, and further deep results have been obtained, for example [**PP**, **K**]. Especially, on classification of subfactors of the approximately finite dimensional (AFD) II_1 factor, A. Ocneanu announced a striking result with the notion of paragroups [**O1**, **O2**]. But the details of his proof have not appeared yet.

Ocneanu's theory has two aspects. One is analytic aspect, which is covered by Popa's deep results [**P1**, **P2**], and the other is combinatorial aspect i.e. the theory of paragroups. Until now, existence and non-existence results of paragroups corresponding to the Coxeter graphs except E_8 have been obtained [**B**, **K**, **SV**, **I**]. The purpose of this paper is to prove the existence of the E_8 paragroup, which shows that Ocneanu's classification list in [**O1**] is correct.

The contents of this paper are as follows. In Section 2 we will show that the study of flat connections on E_8 is reduced to that of other connections on some four graphs, two of which are E_8. In Section 3 we will prove the main result by computing the above-mentioned connections. While we will treat only E_8 case our method is applicable to the other cases of the Coxeter graphs. Throughout this paper we will freely use the contents and the notations in [**K**].

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