PACIFIC JOURNAL OF MATHEMATICS
Vol. 166, No. 2, 1994

THE NUMBER OF LATTICE POINTS WITHIN A CONTOUR AND VISIBLE FROM THE ORIGIN

DOUG HENSLEY

The main result is an estimate for the number $P(r)$ of relatively prime pairs (a, b) of integers within a contour. When specialized to the contour $x^2 + y^2 = r$ this estimate gives

$$P(r) = (6/\pi)r + \left(\text{ without the RH, } O_\epsilon\left(r^{1/2}\exp(-(\log r)^{(3/5)+\epsilon})\right)\right. \\ \left.\text{or with the RH } O_\epsilon r^{(51+\epsilon)/100}\right).$$

A similar estimate, with the same sort of error, is obtained for the number of relatively prime pairs (a, b) of positive integers so that $ab \leq r$. The error term for a general contour depends on the maximal value of the radius of curvature of the bounding contour.

1. Introduction. The number $R(r)$ of integer pairs (a, b) for which $a^2 + b^2 \leq r$ is known to satisfy

$$R(r) = \pi r + O_\epsilon(r^{\theta+\epsilon})$$

where $1/4 \leq \theta \leq 7/22$. The main result here is an estimate of the number $P(r)$ of relatively prime integer pairs (a, b) within a contour. When specialized to the contour $x_1^2 + x_2^2 = r$, it gives

$$P(r) = (6/\pi)r + O_\epsilon\left(r^{1/2}\exp\left(-(\log r)^{(3/5)+\epsilon}\right)\right).$$

If the Riemann Hypothesis (RH) holds, this improves to

$$P(r) = (6/\pi)r + O_\epsilon\left(r^{(51/110)+\epsilon}\right).$$

If we assume further that the correct value of θ in the circle problem is 1/4, then the exponent 51/110 becomes 9/10. We give a