PACIFIC JOURNAL OF MATHEMATICS
Vol. 166, No. 2, 1994

THE CAUCHY INTEGRAL, ANALYTIC CAPACITY AND SUBSETS OF QUASICIRCLES

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In this paper, we show that if $K \subset \mathbb{C}$ is AD-regular and sufficiently flat, then K is a subset of a chord-arc curve if the Cauchy integral operator is bounded on $L^2(K)$. This result partially answers a question raised by G. David, P. Jones and S. Semmes. Also, we prove that if K is as above (locally) and has positive analytic capacity, then K must contain a subset of a rectifiable curve of positive length. Finally, we characterize subsets of some quasicircles in terms of a simple geometric condition invented by P. Jones.

Introduction. Let $K \subset \mathbb{C}$ be a bounded set and, for $\delta > 0$, write

$$\Lambda^\delta(K) = \inf\left\{ \sum_{j=1}^{\infty} \delta_j : \ K \subset \bigcup_{j=1}^{\infty} D(a_j, \delta_j); \ \delta_j \leq \delta \right\},$$

where $D(a_j, \delta_j) = \{ z : |z - a_j| \leq \delta_j \}$. Then $\Lambda^\delta(K)$ is a decreasing function of δ. The one-dimensional Hausdorff measure $\Lambda(\cdot)$ is defined by

$$\Lambda(K) = \lim_{\delta \to 0} \Lambda^\delta(K).$$

If K is connected and $\Lambda(K) < \infty$, then we call it a rectifiable curve.

DEFINITION 1. A Λ-measurable set $K \subset \mathbb{C}$ is said to be regular in the sense of Ahlfors and David, or AD-regular, if there exists $M < \infty$ such that for all $x \in K$ and $0 < r \leq \operatorname{diam}(K)$,

$$M^{-1} r \leq \Lambda(K \cap D(x, r)) \leq Mr.$$

If K is connected we call it an AD-regular curve.