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MULTIPLIERS BETWEEN INVARIANT SUBSPACES OF THE BACKWARD SHIFT

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Contained in the Hardy space H^2 on the unit disk in the complex plane are certain Hilbert spaces which are invariant under the adjoint of the shift. One such space $\mathcal{H}(b)$ is associated with each function b in the closed unit ball of H^∞. In the special case where b is an inner function, $\mathcal{H}(b)$ is just the subspace of H^2 orthogonal to the shift-invariant subspace bH^2. It is proven here that for any functions b_1 and b_2 in the closed ball of H^∞, the spaces $\mathcal{H}(b_1)$ and $\mathcal{H}(b_2)$ are isometrically isomorphic under a multiplication operator if and only if there is a disk automorphism τ such that $b_2 = \tau \circ b_1$. In this case, the multiplicative isomorphism is determined explicitly and uniquely. This motivates an investigation of *multipliers between* $\mathcal{H}(b_1)$ and $\mathcal{H}(b_2)$, that is, multiplication operators acting bijectively but not necessarily isometrically. Restricting to the case where b_1 and b_2 are inner functions, it is shown that a multiplier between given spaces is unique up to multiplication by a nonzero constant, and several theorems are proven concerning the existence of such multipliers. Finally, consideration is given to the implications of these results for the characterization of the invariant subspaces in H^2 on an annulus.

1. Introduction. To any analytic function f on the unit disk can be applied the mappings $f \mapsto zf$ and $f \mapsto z^{-1}[f - f(0)]$. On the Hilbert space H^2 these mappings are adjoint operators called, respectively, the shift S and the backward shift S^*. For convenience the notation S and S^* will be used even when the mappings are applied to functions not in H^2. If f and g are analytic functions on the disk, then $S^*(fg) = fS^*g + g(0)S^*f$. This simple algebraic identity will be applied repeatedly without comment.