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GEOMETRIC ASPECTS OF BÄCKLUND TRANSFORMATIONS OF WEINGARTEN SUBMANIFOLDS

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If f_1 and f_2 are immersions of an n-manifold M into $\mathbf{R}^{2n-1}$ such that their induced frame bundles differ by a constant right action, then f_1 and f_2 both satisfy a Weingarten condition on their normal bundles and the right action corresponds to a generalization of the classical Bäcklund transformation.

1. Introduction. In the 1890s Bianchi, Lie, and finally Bäcklund looked at what are now called Bäcklund transformations of surfaces. In modern parlance, they begin with two surfaces in Euclidean space in a line congruence: there is a mapping between the surfaces M_1 and M_2 such that the line through any two corresponding points is tangent to both surfaces. Bäcklund proved that if a line congruence satisfied two additional conditions, that the line segment joining corresponding points has constant length, and that the normals at corresponding points form a constant angle, then the two surfaces are necessarily surfaces of constant negative curvature. He was also able to show that a Bäcklund transformation is integrable, in the sense that given a point on a surface of constant negative curvature and a tangent line segment at that point, a new surface of constant negative curvature can be found, containing the endpoint of the line segment, that is a Bäcklund transform of the original surface.

Since that time, much of the attention has focused on Bäcklund transformations as transformations of solutions to partial differential equations. Since a surface of constant negative curvature is equivalent to a solution of the Sine-Gordon equation, the Bäcklund