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A NON-HAKEN HYPERBOLIC 3-MANIFOLD COVERED BY A SURFACE BUNDLE

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The question as to whether a finite volume hyperbolic 3-manifold has a finite cover which fibers over the circle, seems, at present, completely mysterious. In this paper we give the first explicit examples of non-Haken hyperbolic 3-manifolds covered by a manifold that fibers of the circle. The methods used are arithmetic using the theory of quaternion algebras.

1. Introduction. One of the outstanding unsolved questions in the theory of hyperbolic 3-manifolds is whether every closed hyperbolic 3-manifold has finite cover with positive first betti number. A much stronger question of Thurston (see [**28**, Question 18]), which if answered affirmatively would imply an affirmative solution to the previous question is whether every finite volume hyperbolic 3-manifold has finite cover which fibers over the circle, the fiber being a compact surface possibly with punctures. There is significant evidence in the first case to support that the conjectured answer to the first question is "yes". However the second situation still seems, at present, completely mysterious.

There is a simple way to construct (closed) Haken hyperbolic 3-manifolds which do not fiber over the circle, but have a double cover which does. Namely one can form the union of two twisted I-bundles over a non-orientable surface (see [**10**, Chapter 10] for definitions). By unwrapping the I-bundle in a double cover, one obtains a manifold which fibers over the circle, see [**10**, Chapter 11]. It is not hard to control that the monodromy in the double cover be pseudo-Anosov, and hence the manifolds are hyperbolic by [**29**]. (See Theorem 2 for some specific examples.)