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LOCAL REPRODUCING KERNELS ON WEDGE-LIKE DOMAINS WITH TYPE 2 EDGES

AL BOGGESS AND ALEXANDER NAGEL

We represent holomorphic functions on a wedge-like domain by positive integral kernels which are defined on the edge of the wedge. Type 2 edges are considered. As an application, we show that an H^p function on a wedge has pointwise almost everywhere limits on the edge within admissable approach regions in the wedge.

A striking fact about function theory in several variables is that, under suitable convexity hypotheses on a domain $\Omega \subset \mathbb{C}^n$ with $n > 1$, if $z_0 \in \Omega$ is "close" to the boundary $\partial\Omega$, then there are representing measures for z_0 whose support on $\partial\Omega$ is compactly supported and "close" to z_0^*, the projection of z_0 onto the boundary. This is false for domains in $\mathbb{C}$, for general non-convex domains in $\mathbb{C}^n$, and for harmonic functions on domains in $\mathbb{R}^{2n}$.

We illustrate this phenomenon with a simple example. Let $\Omega = \{(z,w) \in \mathbb{C}^2; \mathrm{Re}(z) > |w|^2\}$. The boundary of Ω will be denoted by $\Sigma = \{(z,w) \in \mathbb{C}^2; \mathrm{Re}(z) = |w|^2\}$ and can be identified with the Heisenberg group. We wish to represent the value of a holomorphic function F on Ω at the point $(0,r) \in \Omega$ for $r > 0$, by integrating F against a suitable measure on Σ. To do this, we let $\phi \in C_0^\infty(\mathbb{C})$ be a radial function with support in the unit disc and whose integral over $\mathbb{C}$ is one. Let

$$\phi_r(w) = \frac{4}{r^2}\phi\left(\frac{2(w-r)}{r}\right).$$

The function ϕ_r has support in the disc centered at r with radius $r/2$ and the integral of ϕ_r over $\mathbb{C}$ is one. The mean value property for holomorphic functions shows that

$$F(0,r) = \iint_{w\in\mathbb{C}} F(0,w)\phi_r(w)\,dx\,dy$$