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DETERMINING MULTIPLICITIES OF HALF-INTEGRAL WEIGHT NEWFORMS

THOMAS R. SHEMANSKE AND LYNNE H. WALLING

We decompose the space of newforms of half-integral weight into a direct sum of spaces of newforms of integral weight which occur with multiplicity one or two. This not only demonstrates in a precise way the failure of a multiplicity-one result to hold for half-integral weight newforms, but moreover indicates which spaces occur with a given multiplicity. The spaces occuring with multiplicity two are shown to be in one-to-one correspondence with a collection of Kohnen subspaces. As a consequence, it is shown that under the Shimura correspondence, the level a newform of half-integral weight is not determined by the level of the integral weight newform to which it corresponds.

Since a knowledge of the Hecke eigenvalues is insufficient to charectesize half-integral weight newforms up to a scalar, we develop sufficient conditions on the squarefree coefficients, augmenting the information on the eigenvalues, which allow such a characterization. In the last section, these later results are carried over to the Hilbert modular setting.

Introduction. It is well-known that newforms of half-integral weight do not satisfy a multiplicity-one theorem, that is in general, they cannot be characterized solely by their Hecke eigenvalues ([**5**]). In one attempt to deal with this difficulty, Kohnen [**6**] defines a subspace of the space of half-integral weight cusp forms which does have a newform theory in which the newforms satisfy a multiplicity-one theorem. On the other hand, he develops this theory only for cusp forms having restricted Fourier expansions as well as squarefree level and quadratic character.

In [**16**, **17**], Ueda significantly extends Kohnen's theory by establishing trace identities which relate the trace of Hecke operators on various spaces of cusp forms of integral and half-integral weight. By using these trace identities, Ueda is able to give a decomposition