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S-INTEGER POINTS ON ELLIPTIC CURVES

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We give a quantitative bound for the number of S-integral points on an elliptic curve over a number field K in terms of the number of primes dividing the denominator of the j-invariant, the degree $[K : \mathbb{Q}]$, and the number of primes in S.

Let K be a number field of degree d and M_K the set of places of K. Let E/K be an elliptic curve with quasi-minimal Weierstrass equation

$$E \quad : \quad y^2 = x^3 + Ax + B.$$

If $\Delta = 4A^3 + 27B^2$ is the discriminant of this equation, recall that quasi-minimal means that $|N_{K/\mathbb{Q}}(\Delta)|$ is minimized subject to the condition that $A, B \in \mathrm{O}_K$. Let $S \subset M_K$ be a finite set of s places containing all the archimedean ones, and denote the ring of S-integers by O_S. Let j be the j-invariant of E.

In [**11**], Silverman proved that if j is integral, then

$$\#\{P \in E(K) : x(P) \in \mathrm{O}_S\}$$

can be bounded in terms of the field K, $\#S$, and the rank of $E(K)$. More generally, Silverman proved that if the j-invariant is non-integral for at most δ places of K, then that set can be bounded in terms of the previously mentioned constants and δ. This is a special case of a conjecture of Lang asserting the existence of such a bound which is independent of δ. However, Silverman did not explicitly compute the constants involved.

In this paper, using more explicit methods, we compute the dependence of the bounds on the various constants. In particular, as a consequence of Proposition 11, we have the following

THEOREM 0.1. *For elliptic curves E/K of sufficiently large height, the number of S-integral points is at most* $2{\cdot}10^{11}d\delta(j)^{3d}(32{\cdot}10^9)^{r\delta(j)+s}$.