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HARDY SPACES AND OSCILLATORY SINGULAR INTEGRALS: II

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We consider oscillatory singular integral operators with real-analytic phases. The uniform boundedness from $H^1_E \to L^1$ of such operators is proved, where H^1_E is a variant of the standard Hardy space H^1. The result is false for general C^∞ phases. This work is a continuation of earlier work by Phong and Stein (on bilinear phases) and the author (on polynomial phases).

1. Introduction. In [**8**], Phong and Stein established an H^1 theory for oscillatory singular integral operators with bilinear phase functions. Their H^1 boundedness result, together with the L^2 estimate for such operators, led to the L^p boundedness of oscillatory singular integral operators with bilinear phases, via interpolation.

Ricci and Stein considered oscillatory singular integral operators with polynomial phases. They showed that such operators are bounded on L^p for $1 < p < \infty$, and the bound for the operator norm depends only on the degree of the polynomial, not its coefficients ([**9**]). In [**1**], Chanillo and Christ proved that such operators are of weak-type (1,1).

In an earlier paper, we extended Phong-Stein's H^1 theory for operators with bilinear phases to operators with polynomial phases. Let $x, y \in \mathbb{R}^n$, $K(x, y)$ be a Calderón-Zygmund kernel, $P(x, y)$ be a real-valued polynomial in x and y. Define T:

$$Tf(x) = \text{p. v.} \int_{\mathbb{R}^n} e^{iP(x,y)} K(x, y) f(y) dy. \tag{1.1}$$

The following theorem is proved in [**4**].

THEOREM A. *The operator T is bounded from H^1_E to L^1, and the bound for $\|T\|$ depends only on the degree of P, not its coefficients.*

The space H^1_E in Theorem A depends on P and is an variant of the standard Hardy space H^1. The precise definition for H^1_E will be