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THE L^p THEORY OF STANDARD HOMOMORPHISMS

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Suppose that $\phi : L^1(\omega_1) \to L^1(\omega_2)$ is a continuous nonzero homomorphism between weighted convolution algebras on R^+, and let ϕ also designate the extension of this map to the corresponding measure algebras $M(\omega_1)$ and $M(\omega_2)$. For $1 < p < \infty$, we prove: (a) the semigroup $\mu_t = \phi(\delta_t)$ acts as a strongly continuous semigroup on $L^p(\omega_2)$; (b) Whenever $L^1(\omega_1) * f$ is dense in $L^1(\omega_1)$, then $L^p(\omega_2) * \phi(f)$ is dense in $L^p(\omega_2)$; (c) Each h in $L^p(\omega_2)$ can be factored as $h = \phi(f) * g$; (d) ϕ is continuous from the strong operator topology of $M(\omega_1)$ acting on $L^1(\omega_1)$ to the strong operator topology of $M(\omega_2)$ acting on $L^p(\omega_2)$.

1. Introduction. In this paper we show that the L^p analogue of a number of questions we have studied ([**10**], [**8**], [**11**], [**7**]) involving homomorphisms and semigroups on weighted L^1 spaces on $R^+ = [0, \infty)$ all have positive answers when $1 < p < \infty$. If $\omega(t) > 0$ is a Borel function on R^+ which is locally bounded and locally bounded away from 0 and if $1 \leq p < \infty$, we let $L^p(\omega)$ be the Banach space of (equivalence classes of) measurable functions on R^+ with $f\omega$ in $L^p(R^+)$, with the inherited norm

$$\|f\| = \|f\|_{\omega,p} = \|f\omega\|_p = \left(\int_0^\infty |f(t)\omega(t)|^p\, dt\right)^{1/p}.$$

We are particularly interested in the case that $L^1(\omega)$ is a Banach algebra and all $L^p(\omega)$ are $L^1(\omega)$-modules under the usual convolution multiplication $f * g(x) = \int_0^x f(x-t)g(t)\, dt$. Therefore we will usually assume that $\omega(t)$ is an *algebra weight*, that is $\omega(t)$ satisfies:

(1) $\omega(x+y) \leq \omega(x)\omega(y)$;
(2) $\omega(x)$ is right continuous;
(3) $\omega(0) = 1$.

(1), (2), and (3) are just normalizations and are essentially equivalent to $L^1(\omega)$ being an algebra in which case $L^p(\omega)$ is an $L^1(\omega)$-module [**9**], where the module action is convolution. The most