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APPLICATIONS OF SUBORDINATION CHAINS TO STARLIKE MAPPINGS IN $\mathbb{C}^n$

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We use the work of Pfaltzgraff on subordination chains in $\mathbb{C}^n$ to recover a growth theorem for starlike mappings of the unit ball established recently by Barnard, FitzGerald and Gong. We also introduce a class of *strongly starlike* maps for which we construct, aided by the aforementioned technique, an explicit quasiconformal extension to $\mathbb{C}^n$. Several examples are discussed at the end.

1. Introduction. Let f be a univalent map of the unit disc, with $f(0) = 0$ and $f'(0) = 1$. The celebrated Koebe theorem asserts that the image of f contains a disc about the origin of radius 1/4, 1/4 being sharp. This theorem has no analogue in several complex variables, whether one deals with normalized univalent maps of the unit ball B^n or the polydisc. By normalized we mean fixing the origin and having the identity as differential at that point. In particular, the classical growth theorem in dimension 1

$$\frac{|z|}{(1+|z|)^2} \le |f(z)| \le \frac{|z|}{(1-|z|)^2} \tag{1.1}$$

is no longer valid in higher dimensions. Remarkably, (1.1) persists for arbitrary n when considering the subclass of *starlike* maps of B^n, as Barnard, FitzGerald and Gong have recently shown [**BFG 1**]. The result is sharp. Recall that a map is called starlike if the image is starlike with respect to the origin. Suffridge has given the following alternative local characterization: let $w(z) = (Df)^{-1}(f)$, where the differential and the function are evaluated at z. Then f is starlike if and only if

$$\mathrm{Re}\langle \bar{z}, w(z)\rangle \ge 0\,. \tag{1.2}$$

Here $\langle a, b\rangle = \sum a_i b_i$ for $a, b \in \mathbb{C}^n$ [**S 1**]. When $n = 1$ then (1.2) recovers the condition $\mathrm{Re}\{z\frac{f'}{f}\} \ge 0$. The proof in [**BFG 1**] uses (1.2)