PACIFIC JOURNAL OF MATHEMATICS
Vol. 168, No. 1, 1995

ON UNIFORM HOMEOMORPHISMS OF THE UNIT SPHERES OF CERTAIN BANACH LATTICES

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We prove that if X is an infinite dimensional Banach lattice with a weak unit then there exists a probability space (Ω, Σ, μ) so that the unit sphere of $(L_1(\Omega, \Sigma, \mu))$ is uniformly homeomorphic to the unit sphere $S(X)$ if and only if X does not contain l_∞^n's uniformly.

1. Introduction. Recently E. Odell and Th. Schlumprecht [**O.S**] proved that if X is an infinite dimensional Banach space with an unconditional basis then the unit sphere of X and the unit sphere of l_1 are uniformly homeomorphic if and only if X does not contain l_∞^n uniformly in n. We extend this result to the setting of Banach lattices. In Theorem 2.1 we obtain that if X is a Banach lattice with a weak unit then there exists a probability space (Ω, Σ, μ) so that the unit sphere $S(L_1(\Omega, \Sigma, \mu))$ is uniformly homeomorphic to the unit sphere $S(X)$ if and only if X does not contain l_∞^n uniformly in n. A consequence of this -Corollary 2.11- is that if X is a separable infinite dimensional Banach lattice then $S(X)$ and $S(l_1)$ are uniformly homeomorphic if and only if X does not contain l_∞^n uniformly in n. Quantitative versions of this corollary are given in Theorem 2.2 and Theorem 2.3. A continuous function $f : [0, \infty) \to [0, \infty)$ with$f(0) = 0$ is a *modulus of continuity* for a function between two metric spaces $F : (A, d_1) \to (B, d_2)$ if $d_2(F(a_1), F(a_2)) \leq f(d_1(a_1, a_2))$ whenever $a_1, a_2 \in A$. Theorem 2.2 says that if X and Y are separable infinite dimensional Banach lattices with $M_q(X) < \infty$ and $M_{q'}(Y) < \infty$ for some $q, q' < \infty$ then there exists a uniform homeomorphism $F : S(X) \to S(Y)$ such that F and F^{-1} have modulus of continuity f where f depends solely on $q, q', M_q(X)$ and $M_{q'}(Y)$. Here $M_q(X)$ is the q-concavity constant of X and will be defined below.

Central in defining these homeomorphisms is the entropy map, considered in [**G**] and [**O.S**]. We refer the reader to [**B**] and its