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COMPACT CONTRACTIBLE n-MANIFOLDS HAVE ARC SPINES ($n \geq 5$)

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The following two theorems were motivated by questions about the existence of disjoint spines in compact contractible manifolds.

THEOREM 1. *Every compact contractible n-manifold* ($n \geq 5$) *is the union of two n-balls along a contractible* $(n-1)$*-dimensional submanifold of their boundaries.*

A compactum X is a *spine* of a compact manifold M if M is homeomorphic to the mapping cylinder of a map from ∂M to X.

THEOREM 2. *Every compact contractible n-manifold* ($n \geq 5$) *has a wild arc spine.*

Also a new proof is given that for $n \geq 6$, every homology $(n-1)$-sphere bounds a compact contractible n-manifold. The implications of arc spines for compact contractible manifolds of dimensions 3 and 4 are discussed in §5. The questions about the existence of disjoint spines in compact contractible manifolds which motivated the preceding theorems are stated in §6.

1. Introduction. Let M be a compact manifold with boundary. A compactum X is a *spine* of M if there is a map $f : \partial M \to X$ and a homeomorphism $h : M \to \mathrm{Cyl}(f)$ such that $h(x) = q((x,0))$ for $x \in \partial M$. Here $\mathrm{Cyl}(f)$ denotes the mapping cylinder of f and $q : (\partial M \times [0,1]) \cup X \to \mathrm{Cyl}(f)$ is the natural quotient map. Thus $q_{|\partial M \times [0,1)}$ and $q|X$ are embeddings and $q(x,1) = q(f(x))$ for $x \in \partial M$. So h carries ∂M homeomorphically onto $q(M \times \{0\})$, $h^{-1} \circ q_{|X}$ embeds in X int M, and $M - h^{-1}(q(X)) \cong \partial M \times [0,1)$.

An arc A in the interior of an n-manifold M is *tame* if A has a neighborhood U in M such that (U, A) is homeomorphic to $(\mathbb{R}^n, [-1,1] \times (0,0\ldots,0))$. An arc in the interior of a manifold is *wild* if it is not tame.