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FOURIER COEFFICIENTS OF AN ORTHOGONAL EISENSTEIN SERIES

JERRY SHURMAN

This paper defines a nonholomorphic Eisenstein series for a totally real algebraic number field F and the special orthogonal group with respect to a bilinear form $S = \begin{pmatrix} T & & \\ & 0 & -1 \\ & -1 & 0 \end{pmatrix}$, where $T \in M_n(F)$ and its embedded images $T^v \in M_n(\mathbb{R})$ under archimedean places v of F have signature $(1, n-1)$. This group has an associated product of tube domains $\mathcal{H}^{\mathbf{a}} = \prod_{v \in \mathbf{a}} \mathcal{H}_v$, the product taken over archimedean places of F and each $\mathcal{H}_v \subset \mathbb{C}^n$. The series is denoted $E(z, s; k, \psi, \mathfrak{b})$ or simply $E(z, s)$, with $z \in \mathcal{H}^{\mathbf{a}}$, $s \in \mathbb{C}$ a complex parameter, $k \in \mathbb{Z}$ the weight, ψ a Hecke character on the ideles of F, and the level $\mathfrak{b}$ an integral ideal in F. E has the Fourier expansion

$$E(z,s) = (-1)^{dk} 2^{d(k+2s)} \sum_{h \in L'} a(h,y,s)\, \mathbf{e}\left(\sum_{v \in \mathbf{a}} T^v(x_v, h_v)\right),$$

where $d = [F : \mathbb{Q}]$, L' is the lattice dual to $\mathfrak{o}_F^n$ under T, $\mathbf{e}(x) = e^{2\pi i x}$, and $z = (x_v + iy_v)_{v \in \mathbf{a}} \in \mathcal{H}^{\mathbf{a}}$. The Fourier coefficient $a(h, y, s)$ is the product $(N\mathfrak{d})^{-\frac{n}{2}} a_{\mathbf{a}}(h, y, s) a_{\mathbf{f}}(h, s)$ with $N\mathfrak{d}$ the norm of the different of F over $\mathbb{Q}$. The archimedean factor is $a_{\mathbf{a}}(h, y, s) = \prod_{v \in \mathbf{a}} \xi(y_v, h_v; k + s, s; T^v)$ with ξ a certain confluent hypergeometric function studied by Shimura. The nonarchimedean factor $a_{\mathbf{f}}(h, s)$ is essentially a product and quotient of Hecke L-functions, depending on the parity of n and the nature of h. Specializing to $s = 0$ gives holomorphic and in special cases nearly holomorphic behavior.

1. Introduction and notation.

Introduction. This paper defines an Eisenstein series $E(z, s)$ of weight k for z in a tube domain and s a complex parameter, and computes its Fourier expansion explicitly. The series is of interest as a special case of the nearly holomorphic functions studied by Shimura and Bluher.