PACIFIC JOURNAL OF MATHEMATICS
Vol. 168, No. 2, 1995

THE SPHERICAL MEAN VALUE OPERATOR FOR COMPACT SYMMETRIC SPACES

V. PATI, M. SHAHSHAHANI AND A. SITARAM

When M is a compact symmetric space, the spherical mean value operator L_r (for a fixed $r > 0$) acting on $L^2(M)$ is considered. The eigenvalues λ for $L_r f = \lambda f$ are explicitly determined in terms of the elementary spherical functions associated with the symmetric space. Alternative proofs are also provided for some results of T. Sunada regarding the special eigenvalues +1 and −1 using a purely harmonic analytic point of view.

1. Introduction. In a series of papers ([**Su1**, **Su2**, **Su3**]) T. Sunada has considered (among other things) the "spherical mean operator" of a fixed radius r on a compact Riemannian manifold Y and has examined its connections with the so-called 'Geodesic Random Walk' problem. If $r > 0$, the spherical mean operator L_r is defined on $L^2(Y)$ by:

$$(L_r f)(x) = \int_{\{X \in T_x(Y): \|X\|=1\}} f(\operatorname{Exp}_x rX)\, d\sigma(X).$$

(Here $T_x(Y)$ is the tangent space at $x \in Y$ equipped with the inner product arising from the Riemannian structure, Exp_x the exponential map from $T_x(Y)$ into Y and $d\sigma$ the normalized measure on the surface of the unit sphere in $T_x(Y)$.) Roughly speaking $(L_r f)(x)$ is the mean value of f at a geodesic distance r from x. This note grew out of an attempt to understand the results of Sunada from a group theoretic/harmonic analytic point of view. In fact we show that for symmetric spaces of the compact type the ergodicity and eigenvalue problems considered in [**Su1**] are consequences of simple and elementary arguments (Propositions 2.4 and 2.5). This point of view also sheds some light on the difference between spheres, symmetric spaces of rank 1 and higher rank spaces.