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# GENERALIZED WAHL MAPS AND ADJOINT LINE BUNDLES ON A GENERAL CURVE

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**For any two line bundles on a smooth curve, there are so called Wahl maps, that can be viewed as generalizations of the ordinary Gaussian. These maps govern various properties of the projective embeddings of $C$, like for example the first order deformations of the projective cone that smooth the vertex. In this paper we investigate these maps from the point of view of the intrinsic geometry of $C$, by applying an approach of Voisin for the case $L = N = K$.**

**1. Introduction.** Consider a smooth projective curve $C$ and two line bundles $L$ and $N$ on it. It is well known that there is a linear map, given by section multiplication

$$\mu : H^0(C, L) \otimes H^0(C, N) \longrightarrow H^0(C, L + N).$$

We define the *module of relations* of $L$ and $N$, denoted $R(L, N)$, to be the kernel of $\mu$. The Wahl map, or Gaussian map

$$\gamma_{L,N} : R(L, N) \longrightarrow H^0(K + L + N)$$

(where $K$ denotes the canonical line bundle on $C$) is defined by making sense of the expression $\gamma_{L,N}(s, t) =: sdt - tds$. These maps have attracted increasing attention since Wahl's basic observation that they relate to the deformation theory of the projective cone over $C$ ([**W88**]). In fact, if $L$ is a very ample line bundle on $C$ the cokernels of $\gamma_{K,L^{i-1}}$, for $i$ positive, are dual to the first order deformations of the projective cone which smooth the vertex. From this it follows, for example, that if $C$ is the hyperplane section of a (projective) $K3$ surface, then $\gamma_{K,K}$ is not surjective. This was proved from a deformation theoretic point of view by Wahl, and along different lines by Beauville and Merindol ([**BM87**]).