PACIFIC JOURNAL OF MATHEMATICS
Vol. 168, No. 2, 1995

A DIFFERENTIABLE STRUCTURE FOR A BUNDLE OF C^*-ALGEBRAS ASSOCIATED WITH A DYNAMICAL SYSTEM

MOTO O'UCHI

Let (M, G) be a differentiable dynamical system, and σ be a transverse action for (M, G). We have a differentiable bundle (B, π, M, C) of C^*-algebras with respect to a flat family $\mathcal{F}_\sigma$ of local coordinate systems and we have a flat connection ∇ in B. If G is connected, the bundle B is a disjoint union of $\rho_x(C_r^*(\mathcal{G}))$ $(x \in M)$, where $\mathcal{G}$ is the groupoid associated with (M, G) and ρ_x is the regular representation of $C_r^*(\mathcal{G})$. We show that, for $f \in C_c^\infty(\mathcal{G})$, a cross section $cs(f) : x \mapsto \rho_x(f)$ is differentiable with respect to the norm topology, and calculate a covariant derivative $\nabla(cs(f))$. Though B is homeomorphic to the trivial bundle, the differentiable structure for B is not trivial in general. Let B^σ be a subbundle of B generated by elements f with the property $\nabla(cs(f)) = 0$. We show the triviality of the differentiable structure for B^σ induced from that for B when $C_r^*(\mathcal{G})$ is simple. We have a bundle $RM(B)$ of right multiplier algebras and it contains B as a subbundle. Let (M, G) be a Kronecker dynamical system and σ be a flow whose slope is rational. In this case, we have a subbundle D of $RM(B)$ whose fibers are $*$- isomorphic to $C(\mathbb{T})$. The flat connection ∇^r in D is not trivial and the bundle B decomposes into the trivial bundle B^σ and the non-trivial bundle D. Moreover, for a σ-invariant closed connected submanifold N of M with $\dim N = 1$, we show that $C_r^*(\mathcal{G}|N)$ is $*$-isomorphic to $C_r^*(D_x, \Phi_x)$, where Φ_x is the holonomy group of ∇^r with reference point x. If G is not connected, we also have sufficiently many differentiable cross sections of B and calculate their covariant derivatives.

0. Introduction. In the theory of C^*-algebras, one sometimes study a stable C^*-algebra $A \otimes \mathcal{K}$ instead of studying a given C^*-algebra A itself, where $\mathcal{K}$ is the algebra of all compact operators on the infinite dimensional separable Hilbert space. There are many