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LENGTH OF JULIA CURVES

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Let the Fatou set of a meromorphic function f have two components with Denjoy-Wolff points at which f is not transcendental. Then the Julia set J is a "circle/line" or is nowhere rectifiable. In particular if f is rational and J is a Jordan curve then it follows it follows (in the second case) that $\dim(J) > 1$.

Let $F : \mathbb{C} \to \mathbb{C}$ be a meromorphic function. The Julia set $J = J(F)$ is the complement of the set of normality of the iterates F^n. Fatou (1920) proved that if F is rational and if J is a closed Jordan curve then either J is a "circle/line" or it has a dense set of points where there is no tangent. Under various special assumptions on the behaviour of F on J one can say more about the case where J is a closed Jordan curve not equal to a circle/line: Fatou claimed that if "F is expanding on J" then J has Hausdorff 1-measure $\lambda(J) = \infty$. (This was proved by Brolin (1965).) Also if "F is expanding on J" Sullivan (1983) proved that the Hausdorff dimension satisfies $\dim(J) > 1$. Sullivan does this by applying his construction of the conformally invariant measure to Brolin's result. The main result of this paper is that these results hold for arbitrary rational functions.

THEOREM 1. *Let $F : \mathbb{C} \to \mathbb{C}$ be a rational function. Suppose that the Julia set J is a Jordan curve. Then J is a circle/line or $\dim(J) > 1$.*

REMARKS. (i) Actually Przytcki, Urbanski and Zdunik [**14**] had proved Theorem 1 for the "repellant case". Also Zdunik [**17**] proves that if F is any polynomial with Julia set J a Jordan curve then $\dim(J) > 1$. We only prove $\lambda(J) = \infty$ and then make good use of Decker and Urbanski [**5**].

(ii) Note that we require J to be a closed Jordan curve. Our result is considerably more general than for Jordan curves. We only require that F has two foreward invariant (disjoint simply connected)