

CONTROLLING TIETZE-URYSOHN EXTENSIONS

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This paper explores some of the possibilities for controlling properties of continuous extensions of continuous functions. For example, particular attention is paid to the problem of preserving some desirable common property (e.g. pairwise disjointness, partition of unity, etc.) of a collection of functions, when the entire collection is extended simultaneously and continuously from a closed subset A of a normal topological space X to the whole space. The functions treated here are mainly real-valued, but in the last section, a procedure introduced by Dugundji is used to show how to preserve the equicontinuity of a collection of functions whose ranges lie in a locally convex metric linear space.

0. Introduction. If A is a closed subset of a normal topological space X , and if $f : A \rightarrow \mathbf{R}$ is continuous function, then by the extension theorems of Tietze [T] and Urysohn [U] (the combination of which we will refer to as [TU]), there exists a continuous extension \hat{f} of f to all of X . Several generalizations of these results have been obtained, notably Katětov's result [K] for uniformly continuous functions on uniform spaces, and Dugundji's result [D] for continuous functions into linear topological spaces. In this article, we present some results which deal with the degree of control we have over the extensions of continuous functions; that is, how we may choose the sets where extended functions take on certain specified values, or satisfy some predetermined relationship.

In Section 1, we show that we may choose, with reasonable topological restrictions, any finite number of disjoint closed subsets of X as level sets for \hat{f} .

We show in Section 2 that if we have a continuous function f and a nonnegative (or nonpositive) continuous function g defined on a closed subset A of a compact metric space X , such that their product fg satisfied $fg = h|_A$ for some continuous function $h : X \rightarrow \mathbf{R}$, then under suitable conditions f and g may be extended