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MULTIPLICATIVE FUNCTIONS ON FREE GROUPS AND IRREDUCIBLE REPRESENTATIONS

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Let Γ be a free group on infinitely many generators. Fix a basis for Γ and for any group element x, denote by $|x|$ its length with respect to this basis. Let e denote the group identity. A *multiplicative function* ϕ on Γ is a function satisfying the conditions $\phi(e) = 1$ and $\phi(xy) = \phi(x)\phi(y)$ whenever $|xy| = |x| + |y|$. We characterize those positive definite multiplicative functions for which the associated representation of Γ is irreducible.

0. Introduction. Fix an infinite set A^+ and let Γ be the free group on generators A^+. Let A^- consist of the the inverse generators and let $A = A^+ \cup A^-$. Any x in Γ admits a unique shortest expression as a product of elements of A. The length of x, $|x|$, is the number of letters in this expression. A *multiplicative* function ϕ on Γ is a function satisfying the conditions

$$\begin{aligned} \phi(xy) &= \phi(x)\phi(y) \qquad \text{when } |xy| = |x| + |y| \\ \phi(e) &= 1 \ . \end{aligned}$$

A multiplicative function is determined by its values on A.

Choose complex numbers $\{\phi(a)\}_{a\in A}$ such that $\phi(a^{-1}) = \overline{\phi(a)}$ and satisfying the condition $\sup_{a\in A} |\phi(a)| < 1$, and extend ϕ to a multiplicative function on Γ. For example, if $0 < r < 1$ one can choose $\phi(a) = r$ for every a. In that case $\phi(x) = r^{|x|}$ is a radial function which Haagerup [**9**] showed to be positive definite. DeMichele and Figà-Talamanca [**4**] extended Haagerup's result, showing that all ϕ constructed as above are positive definite.

To each multiplicative positive definite function ϕ one associates a unitary representation π_ϕ of Γ, specified by the property that π_ϕ has a cyclic vector for which ϕ is the matrix coefficient. When is π_ϕ