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KLEINIAN GROUPS WITH AN INVARIANT JORDAN CURVE: J-GROUPS

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We study some topological and analytical properties of a Kleinian group G for which there is an invariant Jordan curve by the action of G. These groups may be infinitely generated.

1. Preliminaries. A Kleinian group G is a group of Möbius transformations acting discontinuously on some region of the Riemann sphere $\hat{\mathbb{C}}$. The (open) set of points at which G acts discontinuously is called the region of discontinuity of G and it is denoted by $\Omega(G)$. The complement (on the Riemann sphere) of $\Omega(G)$ is called the limit set of G and it is denoted by $\Lambda(G)$. We will denote the upper and lower half-planes by $U = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$ and $L = \{z \in \mathbb{C} : \text{Im}(z) < 0\}$, respectively. If G is a group of Möbius transformations and D is a subset of the Riemann sphere, then the stabilizer of D is defined by $G_D = \{g \in G : g(D) = D\}$.

J-GROUPS. A Kleinian group G is called a J-group if there exists a Jordan curve γ so that $G_\gamma = G$. We say that γ is a $G-$invariant Jordan curve. In this case, we have necessarily that $\Lambda(G) \subset \gamma$. If $\Lambda(G) = \gamma$, then we say that G is a J-group of the first kind; otherwise, we say that it is of the second kind.

If G is a J-group and γ is a $G-$invariant Jordan curve, then we can associate to G a 3-tuple (γ, D_1, D_2), where D_1 and D_2 are the two topological discs bounded by γ. If the J-group G is of the first kind, then such a 3-tuple is unique (modulo permutation of the two topological discs). In any case, the stabilizers G_{D_1} and G_{D_2} coincide. We have that G_{D_1} is either G or it is a subgroup of index two in G. Classical examples of J-groups are given by Fuchsian groups, $\mathbb{Z}_2-$extension of Fuchsian groups, Schottky groups and QuasiFuchsian groups.