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ON BANACH SPACES Y FOR WHICH $B(C(\Omega), Y) = K(C(\Omega), Y)$

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Let Ω be a compact Hausdorff space. In this paper we give some necessary conditions and some sufficient conditions on a Banach space Y in order that all continuous linear operators from $C(\Omega)$ into Y are compact. We prove that for a nonscattered compact Hausdorff space Ω, for Y belonging to a large class of Banach spaces all operators from $C(\Omega)$ into Y are compact if and only if all operators from l^2 into Y are compact.

Introduction. In this paper by the word "operator" we will mean a "continuous linear operator." E. Dubinsky, A. Pelczynski, and H.P. Rosenthal [**8**] have given a characterization of all Banach spaces Y for which all operators from $\mathcal{L}_\infty$ into Y are absolutely 2-summing. Here, our aim is to characterize all Banach spaces Y for which all operators from a $C(\Omega)$-space into Y are compact. We noticed that such a characterization depends on whether the compact Hausdorff space Ω is scattered (dispersed) or nonscattered (nondispersed). So we consider two cases separately.

Case 1: Ω is an infinite scattered compact Hausdorff space. In this case, from some known results we deduce that *all operators from $C(\Omega)$ into a Banach space Y are compact if and only if all operators from a closed subspace of c_0 into Y are compact if and only if Y does not contain a copy of c_0.*

Case 2: Ω is a nonscattered compact Hausdorff space. In this case, we present a necessary condition on a Banach space Y for all operators from $C(\Omega)$ into Y to be compact. Specifically, *if each operator from $C(\Omega)$ into Y is compact, then each operator from l^2 into Y is compact.* Consequently, *for a Banach space Y for which each operator from $C(\Omega)$ into Y is absolutely 2-summing, each operator from $C(\Omega)$ into Y is compact if and only if each operator from l^2 into Y is compact.* Another necessary condition is given by