CLASSES OF MATRICES AND QUADRATIC FIELDS

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1. Introduction. In a recent paper [1] a correspondence between classes of matrices with rational integral elements and ideal classes in algebraic number fields was discussed. This is now studied in more detail in the case of quadratic fields. In particular the ideal classes of order 2 are discussed and the significance of the sign of the norm of the fundamental unit in real quadratic fields is displayed in an example; further results in this connection will be published elsewhere.

For completeness the result of [1] is repeated:

Let $f(x) = 0$ be an irreducible algebraic equation of degree n with integral coefficients, α one of its algebraic roots, $A = (a_{ik})$ an $n \times n$ matrix with rational integers as elements which satisfies $f(x) = 0$, and S a matrix with rational integers as elements and determinant ± 1. It was shown that the matrix classes $S^{-1}AS$ are in one-to-one correspondence with the ideal classes in the ring generated by α. The correspondence can be expressed in the following way: If $\alpha_1, \cdots, \alpha_n$ is a module base for an ideal in the ring and A the matrix for which

$$\alpha(\alpha_1, \cdots, \alpha_n) = A(\alpha_1, \cdots, \alpha_n) \tag{1}$$

then the ideal class determined by $(\alpha_1, \cdots, \alpha_n)$ corresponds to the matrix class determined by A.

2. Inverse classes. Let m be a square-free positive or negative integer. Consider the quadratic field generated by $m^{1/2}$ or $(1/2)(-1 + m^{1/2})$ according as $m \equiv 2, 3\ (4)$ or $\equiv 1\ (4)$. The first result to be proved is the following.

THEOREM 1. *The inverse of an ideal class corresponds to the class determined by the transpose of the matrix class which corresponds to the ideal class.*

Proof. We treat the two cases separately.

(a) The case $m \equiv 2, 3\,(4)$. Here choose $\alpha = m^{1/2}$. Let α_1, α_2 be a module base for an ideal $\mathfrak{a}$. If

$$\alpha_1 = a + bm^{1/2}\ , \qquad \alpha_2 = c + dm^{1/2}$$

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