SCHLICHT TAYLOR SERIES WHOSE CONVERGENCE ON THE UNIT CIRCLE IS UNIFORM BUT NOT ABSOLUTE

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1. Summary. That a Taylor series which converges uniformly on the unit circle C need not converge absolutely on C was proved by Hardy [2] (see also Landau [4, p. 68]; for a simpler example, see Herzog and Piranian [3, Section 4]). *The present paper exhibits two functions that are schlicht on the closed unit disc, and whose Taylor series converge uniformly but not absolutely on C. Each of the examples satisfies an additional restrictive requirement: the first function has only one singular point on C, and the Taylor series*

$$\sum_{k=0}^{\infty} a_k z^{m_k} \tag{1}$$

of the second function has the property that $\lim(m_{k+1} - m_k) = \infty$.

The condition that (1) represent a schlicht function and converge uniformly but not absolutely on C imposes restrictions on the sequence of exponents $\{m_k\}$. For the condition implies that $\sum_{k=0}^{\infty} m_k |a_k|^2 < \infty$ (see Landau [4, p. 65]); since, by Schwarz's inequality, we have

$$\left(\sum_{k=0}^{\infty} |a_k|\right)^2 \leq \sum_{k=0}^{\infty} m_k |a_k|^2 \cdot \sum_{k=0}^{\infty} 1/m_k,$$

it follows that

$$\sum_{k=0}^{\infty} 1/m_k = \infty. \tag{2}$$

It remains an open question whether the condition implies a restriction on $\{m_k\}$ which is stronger than (2).

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