TOPOLOGIES FOR FUNCTION SPACES

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1. Introduction. Let Z^Y denote the class of continuous functions (or "mappings," or "maps")

$$f: Y \to Z \tag{1.1}$$

of a topological space Y into another Z. A great variety of topologies t may be introduced into Z^Y making it into a topological space $Z^Y(t)$. The topologies we deal with in this paper can be classified by using the notion of "continuous convergence" of directed sets (generalized sequences) f_μ in Z^Y as follows: with no reference to any topology Z^Y, we can say f_μ *converges continuously* (Frink [1]; Kuratowski [2]) to f (f_μ and f are elements of Z^Y) if

$$f_\mu(y_\nu) \to f(y) \tag{1.2}$$

whenever $y_\nu \to y$ in Y. (We use the "$\to$" for convergence as in (1.2), as well as for indicating the domain-range relation as in (1.1). The context prevents confusion.) We can classify the topologies t for Z^Y according as to whether

(1.3) convergence in $Z^Y(t)$ implies continuous convergence

or

(1.4) continuous convergence implies convergence in $Z^Y(t)$.

Certainly there are other topologies possible in Z^Y, but we do not discuss these. There may be a topology t satisfying both (1.3) and (1.4), but if so it is unique; see (5.6).

An apparently different approach to the same classification is suggested by homotopy theory. Beside Y and Z, consider a third space X. For a function g defined on $X \times Y$ with values in Z, we can define $g^*(x)$ mapping X into Z^Y by setting $g^*(x)(y) = g(x,y)$. Then a topology t for Z^Y may be such that, for any X,

(1.5) if g is continuous, then g^* is continuous,

or

(1.6) if g^* is continuous, then g is continuous.

Received May 12, 1950.

Pacific J. Math. 1 (1951), 5-31.