PACIFIC JOURNAL OF MATHEMATICS
Vol. 170 No. 1 1995

ON GORENSTEIN SURFACE SINGULARITIES WITH FUNDAMENTAL GENUS $p_f \geqq 2$ WHICH SATISFY SOME MINIMALITY CONDITIONS

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In this paper we study normal surface singularities whose fundamental genus (:= the arithmetic genus of the fundamental cycle) is equal or greater than 2. For those singularities, we define some minimality conditions, and we study the relation between them. Further we define some sequence of such singularities, which is analogous to elliptic sequence for elliptic singularities. In the case of hypersurface singularities of Brieskorn type, we study some properties of the sequences.

Introduction.

Let $\pi : (\tilde{X}, A) \longrightarrow (X, x)$ be a resolution of a normal surface singularity and, where $\pi^{-1}(x) = A = \bigcup_{i=1}^{n} A_i$ is the irreducible decomposition of the exceptional set A. For a cycle $D = \sum_{i=1}^{n} d_i A_i$ ($d_i \in \mathbb{Z}$) on A, $\chi(D)$ is defined by $\chi(D) = \dim_{\mathbb{C}} H^0(\tilde{X}, \mathcal{O}_D) - \dim_{\mathbb{C}} H^1(\tilde{X}, \mathcal{O}_D)$, where $\mathcal{O}_D = \mathcal{O}_{\tilde{X}}/\mathcal{O}(-D)$. Then

$$\chi(D) = -\frac{1}{2}\left(D^2 + DK_{\tilde{X}}\right), \tag{0.1}$$

where $K_{\tilde{X}}$ is the canonical sheaf (or divisor) on $\tilde{X}$. For any irreducible component A_i, we have

$$K_{\tilde{X}} A_i = -A_i^2 + 2g(A_i) - 2 + 2\delta(A_i) \quad \text{(adjunction formula)}, \tag{0.2}$$

where $g(A_i)$ is the genus of the non-singular model of A_i and $\delta(A_i)$ is the degree of the conductor of A_i (cf. [**7**]). The arithmetic genus of $D \geq 0$ is defined by $p_a(D) = 1 - \chi(D)$. Let Z be the fundamental cycle on A (cf. [**1**]). Then the following three holomorphic invariants of surface singularities are defined by (cf. [**1**], [**7**]),

$$\begin{aligned} p_g &= p_g(X, x) = \dim_{\mathbb{C}} R^1\pi_*\mathcal{O}_{\tilde{X}} && \text{(geometric genus)},\\ p_a &= p_a(X, x) = \max_{D \geq 0} p_a(D) && \text{(arithmetic genus)},\\ p_f &= p_f(X, x) = p_a(Z) && \text{(fundamental genus)}. \end{aligned} \tag{0.3}$$