# THE COHOMOLOGY OF HIGHER-DIMENSIONAL SHIFTS OF FINITE TYPE

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We discuss the cohomology of higher-dimensional shifts of finite type, and prove the following: if a $d$-dimensional shift of finite type $X$ has a rich supply of homoclinic points and a certain specification property, then every Hölder cocycle for shift-action of $\mathbb{Z}^d$ on $X$ with values in a locally compact, second countable group with a doubly invariant metric is Hölder-cohomologous to a homomorphism. The result is illustrated with a number of examples.

## 1. Introduction.

One of many surprising differences between $\mathbb{Z}$-actions and $\mathbb{Z}^d$-actions for $d > 1$ is that certain first cohomology groups of the latter actions may be very small. The first example which came to my attention was due to J.W. Kammeyer ([**Kam1**], [**Kam2**]), who proved that every continuous cocycle for the shift-action of $\mathbb{Z}^d$ on the full $d$-dimensional $k$-shift with values in $\mathbb{Z}/2\mathbb{Z}$ is continuously cohomologous to a homomorphism from $\mathbb{Z}^d$ to $\mathbb{Z}/2\mathbb{Z}$ (the terminology is explained in Section 3). A second instance of this phenomenon appeared in a recent paper by A. Katok and R.J. Spatzier [**KaSp**]: every real-valued Hölder cocycle for an Anosov action of $\mathbb{Z}^d$ on a compact manifold is Hölder-cohomologous to a homomorphism, in complete contrast to the rich supply of nontrivial Hölder cocycles for a single Anosov map. Based on ideas in that paper and the machinery developed in [**Sch1**], [**KS1**], and [**KS3**], it was proved in [**KaS**] that every real-valued Hölder cocycle (or, more generally, every real-valued cocycle with summable variation) for an expansive and (topologically) mixing $\mathbb{Z}^d$-action by automorphisms of a compact, abelian group is Hölder- (or continuously) cohomologous to a homomorphism. Since every expansive $\mathbb{Z}^d$-action by automorphisms of a compact, zero-dimensional, abelian group is topologically conjugate to the shift-action of $\mathbb{Z}^d$ on a shift of finite type, the paper [**KaS**] provides further examples of shifts of finite type with small cohomology groups. The situation is different if one considers continuous cocycles with values in a finite, abelian group: in Example 5.3 below we show that expansive and mixing $\mathbb{Z}^d$-actions by automorphisms of a compact, zero-dimensional, abelian group may have