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ON ALMOST-EVERYWHERE CONVERGENCE OF INVERSE SPHERICAL TRANSFORMS

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Suppose that G/K is a rank one noncompact connected Riemannian symmetric space. We show that if f is a bi-K-invariant square integrable function on G, then its inverse spherical transform converges almost everywhere.

1. Introduction.

Recall the Carleson-Hunt theorem about almost-everywhere convergence of the partial sums of the inverse Fourier transform in one dimension. If we take $1 \leq p \leq 2$ and denote by $\widehat{f}$ the Fourier transform of a function f in $L^p(\mathbb{R})$ then for each $R > 0$ there is the partial sum

$$S_R f(x) := \int_{-R}^{R} \widehat{f}(\xi) e^{ix\xi}\, d\xi. \tag{1}$$

There is also the maximal function

$$S^* f(x) := \sup_{R>0} |S_R f(x)|\,. \tag{2}$$

The Carleson-Hunt Theorem states that if $1 < p \leq 2$ then there is a constant $c_p > 0$ such that

$$\|S^* f\|_p \leq c_p \|f\|_p, \qquad \forall f \in L^p(\mathbb{R}). \tag{3}$$

When this is combined with the fact that the inverse Fourier transform converges everywhere for elements of $C_c^\infty(\mathbb{R})$, a dense subspace of $L^p(\mathbb{R})$, then the almost-everywhere convergence of $\{S_R f(x) : R > 0\}$ follows for all $f \in L^p(\mathbb{R})$. In fact, it suffices to know that there is the weak estimate on the truncated maximal operator for all $y > 0$ and $f \in L^p(\mathbb{R})$,

$$\left|\left\{x : \sup_{R>1} |S_R f(x) - S_1 f(x)| > y\right\}\right| \leq c_p \|f\|_p^p / y^p, \tag{4}$$

and this follows from (3). The inequality (3) has been extended to Hankel transforms by Kanjin [4] and Prestini [6], for an appropriate interval of values for p. In this paper we will be concentrating on the L^2 case.