

THE CORESTRICTION OF VALUED DIVISION ALGEBRAS OVER HENSELIAN FIELDS I

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When L/F is an unramified extension of Henselian fields, we analyze the underlying division algebra cD of the corestriction $\text{cor}_{L/F}(D)$ of a tame division algebra D over L with respect to the unique valuations on cD and D extending the valuations on F and L . We show that the value group of cD lies in the value group of D and for the center of residue division algebra, $Z({}^c\overline{D}) \subseteq \mathcal{N}(Z(\overline{D})/\overline{F})^{1/k}$, where $\mathcal{N}(Z(\overline{D})/\overline{F})$ is the normal closure of $Z(\overline{D})$ over \overline{F} and k is an integer depending on which roots of unity lie in F and L .

Introduction.

For any finite separable extension L/F of fields and any central simple algebra A over L , the corestriction of A is a central simple F -algebra obtained as the fixed point algebra under a Galois group action (cf. [Ri]). This induces the map from the Brauer group $\text{Br}(L)$ to $\text{Br}(F)$ corresponding to the homological corestriction. Though this algebraic corestriction is an important tool in the theory of division algebras, it is actually very hard to work with. To gain a better insight into the behavior of the corestriction, we analyze here the corestriction for valued division algebras over Henselian valued fields, for which there is a well-developed structure theory. We will here concentrate on the case when L is inertial (unramified) over F . In a subsequent paper [H2], we will consider the more general case when L is tame over F , i.e. $\text{char}(\overline{F}) \nmid [L:F]$, where \overline{F} is the residue field of the valuation on F .

For any ring R we write $Z(R)$ and R^* for the center of R and the group of units of R , respectively. We will consider only central simple algebras A finite-dimensional over a field F . By Wedderburn's theorem, $A \cong M_n(D)$, a matrix ring over a division algebra D , which is called the *underlying division algebra* of A .

A valued field (F, v) is called *Henselian* if v extends uniquely to each field algebraic over F . For a nice account for several other characterizations of Henselian valuations, see Ribenboim's paper [Rb]. Recall (e.g. from [W1]) that if D is a central division algebra over a Henselian valued field (F, v) , there exists one and only one valuation on D extending v on F .