

## FINE STRUCTURE OF THE MACKEY MACHINE FOR ACTIONS OF ABELIAN GROUPS WITH CONSTANT MACKEY OBSTRUCTION

SIEGFRIED ECHTERHOFF AND JONATHAN ROSENBERG

Let  $G$  be a locally compact group,  $\omega \in Z^2(G, \mathbb{T})$  a (measurable) multiplier on  $G$ , and denote by  $C^*(G, \omega)$  the twisted group  $C^*$ -algebra of  $G$  defined by  $\omega$ . We are only interested in multipliers up to equivalence, so we always tacitly assume that one is free to vary a multiplier within its cohomology class in  $H^2(G, \mathbb{T})$ . In this paper we are basically concerned with the following two problems: the first is to determine the structure of  $C^*(G, \omega)$ , where  $\omega$  is a type I multiplier on the locally compact abelian group  $G$ , and the second is to describe the crossed product  $A \rtimes_{\alpha} G$  of a continuous-trace  $C^*$ -algebra  $A$  by an action of an abelian group  $G$ , such that the corresponding action of  $G$  on  $\hat{A}$  has constant stabilizer  $N$ , and the Mackey obstruction to extending an irreducible representation  $\rho$  of  $A$  to a covariant representation  $(\rho, U)$  of  $(A, N, \alpha_N)$  is equal to a constant multiplier  $\omega \in H^2(N, \mathbb{T})$  for all  $\rho \in \hat{A}$ . The second of these problems is the obvious starting point for the study of the “fine structure of the Mackey machine”, for actions of abelian groups on continuous-trace algebras with “continuously varying” stabilizers and Mackey obstructions.

In case where all Mackey obstructions are trivial, i.e. if  $\alpha$  is pointwise unitary on the stabilizer subgroup  $N$ , these systems have been investigated extensively in the literature (see for instance [18; 22; 24; 27; 28]), and there are also some results for the case of continuously varying stabilizers [6; 26]. But in recent years there has been almost no progress in the investigation of crossed products with non-trivial Mackey obstructions. However, it turns out that many techniques for the investigation of crossed products by pointwise unitary actions can be used also for the case of non-trivial Mackey obstructions.

Let us explain our results in more detail. In Section 1 we start with investigation of the twisted group algebras  $C^*(G, \omega)$  of an abelian group  $G$  and a multiplier  $\omega \in Z^2(G, \mathbb{T})$ . Recall that  $C^*(G, \omega)$  can be realized in different ways. It is possible to write  $C^*(G, \omega)$  as a completion of  $L^1(G, \omega)$ , the  $L^1$ -algebra of  $G$  with convolution and involution twisted by  $\omega$ , or we can write  $C^*(G, \omega)$  as the Green twisted product  $\mathbb{C} \rtimes_{\tau_{\omega}} G(\omega)$ , where  $G(\omega)$