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EQUISINGULARITY THEORY FOR PLANE CURVES WITH EMBEDDED POINTS

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It is studied the foundations of a theory of equisingularity for plane algebroid or complex analytic curves, reduced everywhere except at one point. A definition of equivalence, or equisingularity, for two such curves, involving resolution by means of quadratic transformations is given as well as several definitions of the concept of equisingular one-parameter family of such singularities are proposed. The theory is related to both the theory of equisingularity for plane, reduced curves and that for ideals with finite support. The different notions of equisingularity for families are compared and a number of examples are presented.

Introduction.

The theory of equisingularity for plane, reduced algebroid curves is classical (cf., e.g., the treatise by Enriques and Chisini [**EC**]), it was renewed and expanded by Zariski (cf.[**Z**] and the bibliography cited there) and continued by other authors (cf., e.g., [**C**]). Basically, one attempts to determine when two singularities are equally complicated, and given a family of singularities to find criteria insuring that all its members are "equally singular". In this planar case, it's remarkable that many seemingly different definitions turn out to be equivalent (cf. e.g., [**T1**, 5.3.1]). For skew curves the situation is considerably more complicated. Anyway, it is reasonable to consider, more generally, curves which are not necesarily reduced (see, e.g., [**A**, p.33]). This seems particularly interesting in the case of skew-curves. In this situation, a reduced (even smooth) curve can degenerate into one with embedded points. In the paper [**BG**] it is systematically considered families of such curves, and the behavior of various numerical invariants of the curves in the family.

In the present article, we intend to study in some detail the case of plane algebroid (or locally analytic) curves, reduced everywhere but at one point (or a finite number of points, in the analytic case). We introduce a notion of "equivalence," completely analogous to that for reduced curves, which requires that the desingularization process of the curves, via quadratic transformations, be the same. In this non-reduced case, that process essentially