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NONEXISTENCE AND INSTABILITY IN THE EXTERIOR DIRICHLET PROBLEM FOR THE MINIMAL SURFACE EQUATION IN THE PLANE

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In this paper we investigate the Dirichlet problem

$$(1) \qquad Mu \equiv (1+|Du|^2)\Delta u - D_iuD_juD_{ij}u = 0 \text{ in } \Omega$$

$$(2) \qquad u = f \text{ on } \partial\Omega$$

in a smooth domain $\Omega \subset \mathbb{R}^2$ for which $\mathbb{R}^2\backslash\Omega$ is bounded. We sharpen previous non-existence results for this exterior Dirichlet problem by showing that even the smallness of the α-Hölder norm, $0 \leq \alpha < \frac{1}{2}$ is not enough for the classical solvability of (1) and (2), not imposing any asymptotical conditions at infinity upon possible solutions. In particular, we explicitely exhibit smooth data f of arbitrary small C^α-norm for which (1), (2) is not solvable in the space $C^0(\overline{\Omega}) \cap C^2(\Omega)$. The key idea of our proof is to replace the original problem (1), (2) on a known domain but with unknown boundary conditions at infinity by the corresponding problem on some unknown (bounded) domain, but with fixed boundary data. By the same method we show the instability of the exterior Dirichlet problem with respect to C^α-small perturbations of the boundary data, $0 \leq \alpha < \frac{1}{2}$, provided that Ω is the complement of a strictly convex set.

1. Introduction.

For a *bounded* domain $\Omega \subset \mathbb{R}^n$ it is well known that the mean-convexivity of $\partial\Omega$ is both necessary and sufficient for the unrestricted solvability of the Dirichlet problem to the n-dimensional minimal surface equation [**JS**]. In two dimensions mean-convexivity, of course, agrees with ordinary convexivity. Existence theorems for unbounded convex domains in arbitrary dimensions have been proved by Massari & Miranda [**MaMi**], excluding the case of a halfspace. The half plane was treated in a recent paper of Collin & Krust [**C-Kr**]. For general bounded, not necessarily mean convex domains one has various existence theorems for (1), (2) under a smallness condition on the data f. Already at the beginning of the century, Korn [**Ko**] and Müntz [**Mü**]