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STRONG APPROXIMATE TRANSITIVITY, POLYNOMIAL GROWTH, AND SPREAD OUT RANDOM WALKS ON LOCALLY COMPACT GROUPS

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We extend to continuous groups our recent results on strongly approximately transitive group actions. We are concerned with locally compact second countable groups and standard Borel G-spaces. A G-space $\mathcal{X}$ with a σ-finite quasiinvariant measure α is called strongly approximately transitive (SAT) if there exists a probability measure $\nu \ll \alpha$ such for every Borel set A with $\alpha(A) \neq 0$ and every $\varepsilon > 0$ one can find $g \in G$ with $\nu(gA) > 1 - \varepsilon$. Examples of SAT G-spaces include boundaries of spread out random walks on G. We prove that when G is compactly generated and has polynomial growth then every standard SAT G-space $(\mathcal{X}, \alpha)$ is necessarily purely atomic; when G is additionally connected, $(\mathcal{X}, \alpha)$ is a singleton. The Choquet-Deny theorem for spread out random walks on G follows as a corollary. For a connected G we establish the equivalence of the following conditions: (a) G has polynomial growth; (b) every standard SAT G-space is a singleton; (c) every SAT homogeneous space of G is a singleton; (d) every homogeneous space of G admits a σ-finite invariant measure; (e) the Choquet-Deny theorem holds for every spread out probability measure on G; (f) the Choquet-Deny theorem holds for every absolutely continuous compactly supported probability measure on G.

1. Introduction.

The aim of this paper is to extend to continuous groups our recent results on strongly approximately transitive actions [**9**].

Let G be a group and $\mathcal{X}$ a Borel G-space with a σ-finite quasiinvariant measure α. We denote by $L^1(\mathcal{X}, \alpha)$ the space of complex measures absolutely continuous with respect to α and by $L^1_1(\mathcal{X}, \alpha) \subseteq L^1(\mathcal{X}, \alpha)$ the subspace of probability measures. For $g \in G$ and $\mu \in L^1(\mathcal{X}, \alpha)$ we write $g\mu$ for the measure $(g\mu)(A) = \mu(g^{-1}A)$. $B(G)$ denotes the space of bounded complex functions on G equipped with the sup norm.

Consider the following properties that a probability measure $\rho \in L^1_1(\mathcal{X}, \alpha)$ might possess: