PACIFIC JOURNAL OF MATHEMATICS
Vol. 170 No. 2 1995

SOME BASIC BILATERAL SUMS AND INTEGRALS

MOURAD E.H. ISMAIL AND MIZAN RAHMAN

By splitting the real line into intervals of unit length a doubly infinite integral of the form $\int_{-\infty}^{\infty} F(q^x)\,dx$, $0 < q < 1$, can clearly be expressed as $\int_0^1 \sum_{n=-\infty}^{\infty} F(q^{x+n})\,dx$, provided F satisfies the appropriate conditions. This simple idea is used to prove Ramanujan's integral analogues of his ${}_1\psi_1$ sum and give a new proof of Askey and Roy's extention of it. Integral analogues of the well-poised ${}_2\psi_2$ sum as well as the very-well-poised ${}_6\psi_6$ sum are also found in a straightforward manner. An extension to a very-well-poised and balanced ${}_8\psi_8$ series is also given. A direct proof of a recent q-beta integral of Ismail and Masson is given.

1. Introduction.

The familiar form of the classical beta integral of Euler is

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}, \tag{1.1}$$

Re $(a,b) > 0$. A less familiar form, obtained by a simple change of variable, is

$$B(a,b) = \int_0^\infty \frac{t^{a-1}\,dt}{(1+t)^{a+b}}. \tag{1.2}$$

There have been many extensions of both these forms, see, for example, Askey [**2–5**], Askey and Roy [**6**], Gasper [**9**, **10**], Rahman and Suslov [**18**], and the references therein. A "curious" extension of (1.2) that was given by Ramanujan [**21**] in 1915 is

$$\int_0^\infty t^{a-1}\,\frac{(-tq^{a+b};q)_\infty}{(-t;q)_\infty}\,dt = \frac{\Gamma(a)\,\Gamma(1-a)}{\Gamma_q(a)\,\Gamma_q(1-a)}\,\frac{\Gamma_q(a)\,\Gamma_q(b)}{\Gamma_q(a+b)}, \tag{1.3}$$

where Re $(a,b) > 0$, $0 < q < 1$, the q-gamma function $\Gamma_q(x)$ is defined by

$$\Gamma_q(x) = \frac{(q;q)_\infty}{(q^x;q)_\infty}\,(1-q)^{1-x}, \quad x \neq 0, -1, -2, \dots, \tag{1.4}$$