PACIFIC JOURNAL OF MATHEMATICS
Vol. 170 No. 2 1995

ON PROPER SURJECTIONS WITH LOCALLY TRIVIAL LERAY SHEAVES

R.J. DAVERMAN AND D.F. SNYDER

Let $f : X \to Y$ be a closed proper surjection whose Leray sheaves are locally constant through a given dimension k. Spectral sequences are used to analyze the cohomological connectivity and manifold properties of Y. Generally, when Y has dimension at most k, it is a cohomology k-manifold over a given principal ideal domain R if and only if $H^q(X, X - f^{-1}y; R)$ is isomorphic to $\mathrm{Hom}_R(H^{q-k}(f^{-1}y; R), R)$ for every $y \in Y$ and $q \leq k$. As a result, if X is an orientable $(n + k)$-manifold, each $f^{-1}y$ has the shape of a closed, connected, orientable n-manifold, and Y is finite dimensional, then Y is a generalized k-manifold. Euler characteristic relationships involving X, Y, and the typical fiber $f^{-1}y$ are derived in case the Leray sheaves of f are locally constant in all dimensions and the range has cohomologically finite type.

The main thrust of this paper is to establish the following:

Theorem . *Let $\mathcal{G}$ be an upper semicontinuous decomposition of an orientable $(n + k)$-manifold into subcompacta having the shape of closed, orientable n-manifolds such that the decomposition space B is finite-dimensional and the Leray sheaf of the decomposition map is locally constant through dimension $k - 1$. Then B is a generalized k-manifold with boundary. If the Leray sheaf is also locally constant in dimension k, then B is a generalized k-manifold.*

Even if the Leray sheaf is constant in all dimensions, such a map need not be an approximate fibration, as Example 5.3 of [**D1**] shows.

The paper is organized into three sections. The first of these merely establishes the terminology and symbolism used in the sequel. It is known by a result of J. Dydak [**Dy**] that B is LC^1; moreover, B is an ANR if, and only if, B is cohomologically locally connected. The second section concentrates on establishing cohomology manifold properties for the image of a closed, proper map provided the Leray sheaf is locally constant through certain dimensions. The third section applies the results of Section 2, the above