PACIFIC JOURNAL OF MATHEMATICS
Vol. 171 No. 1 1995

ELLIPTIC FIBRATIONS ON QUARTIC K3 SURFACES WITH LARGE PICARD NUMBERS

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Let q_1 and q_2 be two binary quartic forms. We consider the diophantine equation $q_1(x,y) = q_2(z,w)$ from the geometric view point. Under a mild condition we prove that the $K3$ surface defined by the above equation admits an elliptic fibration whose Mordell-Weil group over $\mathbf{C}(t)$ has rank at least 12. Next, we choose suitable q_1 and q_2 such that the Mordell-Weil group contains a subgroup of rank 12 defined over $\mathbb{Q}(i)$ and a subgroup of rank 8 defined over $\mathbb{Q}$.

1. Introduction.

In contrast to the arithmetic of algebraic curves, especially elliptic curves, the arithmetic theory of algebraic surfaces has not yet been well understood. In the classification theory, $K3$ surfaces occupy a position similar to that of elliptic curves in algebraic curves. Thus it is natural to expect that $K3$ surfaces will prove us a very interesting arithmetic object to study. As evidence, $K3$ surfaces arise naturally in classical diophantine problems. Since a non-singular quartic surface is a $K3$ surface, Euler's equations

$$x^4 + y^4 = z^4 + w^4$$

and

$$x^4 + y^4 + z^4 = w^4$$

define $K3$ surfaces. Also, finding two different Pythagorean triangles of the same area is in this category of diophantine problems, as it is equivalent to find the integral solutions to the equation

$$xy(x^2 - y^2) = zw(z^2 - w^2).$$

(cf. [**Br1**].) As Euler's second equation suggests, these diophantine problems are very difficult in general (cf. [**E**]). A more detailed study using L-functions has just begun for a very special type of equations, including Euler's equations (cf. [**P-Sw**]).