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HOLOMORPHY TESTS BASED ON CAUCHY'S INTEGRAL FORMULA

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We prove some holomorphy tests based on the Cauchy integral formula for the unit disk, the upper half-plane and the complex plane.

1. Statement of the problems and results.

1.1. The classical Morera problem consists on studying the closed rectifiable curves Γ on the complex plane $\mathbb{C}$ such that any continuous function f on $\mathbb{C}$ satisfying

$$\int_{\Gamma}(f\circ\sigma)(z)\,dz=0, \tag{1.1}$$

for every $\sigma\in M(2)$, is entire. Here $M(2)$ denotes the group of holomorphic rigid motions of $\mathbb{C}$, that is the group of all the mappings of the form $\sigma(z)=\alpha z+\beta$, $\alpha,\beta\in\mathbb{C}$, $|\alpha|=1$.

If Γ is the boundary of a "regular" domain Ω then the above problem is equivalent to the classical Pompeiu problem, *i.e.* when the only continuous function f on $\mathbb{C}$ such that

$$\int_{\Omega}(f\circ\sigma)(z)\,dm(z)=0, \qquad \text{for every } \sigma\in M(2),$$

is $f\equiv 0$.

Note that both problems are invariant under the action of the group $M(2)$, in the sense that if Γ (respectively, Ω) has the Morera (resp., Pompeiu) property then $\sigma(\Gamma)$ (resp., $\sigma(\Omega)$) also does, for every $\sigma\in M(2)$.

A lot of work on those problems has been done by several authors, among them, Brown, Schreiber and Taylor [**8**], Zalcman [**23**], Berenstein [**1**], Berenstein and Yang [**4**], Williams [**21**, **22**], Brown and Kahane [**7**], Garofalo and Segala [**12**, **13**], and Ebenfelt [**10**].

One of the most general results known about the Morera problem is the following: if Γ is a non real-analytic curve which is the boundary of a Jordan Lipschitz domain then Γ has the Morera property.