PACIFIC JOURNAL OF MATHEMATICS
Vol. 171 No. 1 1995

PARTITIONS, VERTEX OPERATOR CONSTRUCTIONS AND MULTI-COMPONENT KP EQUATIONS

M.J. BERGVELT, A.P.E. TEN KROODE

For every partition of a positive integer n in k parts and every point of an infinite Grassmannian we obtain a solution of the k component differential-difference KP hierarchy and a corresponding Baker function. A partition of n also determines a vertex operator construction of the fundamental representations of the infinite matrix algebra gl_∞ and hence a τ function. We use these fundamental representations to study the Gauss decomposition in the infinite matrix group Gl_∞ and to express the Baker function in terms of τ-functions. The reduction to loop algebras is discussed.

1. Introduction.

1.1. Infinite Grassmannians and Hirota equations. Sato discovered that the Kadomtsev–Petviashvili (KP) hierarchy of soliton equations could be interpreted as the Plücker equations for the embedding of a certain infinite Grassmannian in infinite dimensional projective space, see e.g. [**Sa1, Sa2**].

Let us first recall the finite dimensional situation. The Grassmannian $Gr_j(\mathbb{C}^n)$ consists of all j-dimensional subspaces W of the n-dimensional complex linear space $\mathbb{C}^n$. Let $\{e_i \mid i = 1, 2, \dots, n\}$ be a basis for $\mathbb{C}^n$ and let $H_j \in Gr_j(\mathbb{C}^n)$ be the subspace spanned by the first j basis vectors $e_1, e_2, \dots, e_j$. The stabilizer in $Gl(n, \mathbb{C})$ of H_j is the "parabolic" subgroup P_j consisting of invertible matrices $X = \sum X_{ab}E_{ab}$, with $X_{ab} = 0$ if $a > j$ and $b \le j$. Here E_{ab} is the elementary matrix with as only non zero entry a 1 on the $(a, b)^{\text{th}}$ place. So $Gr_j(\mathbb{C}^n)$ can be identified with the homogeneous space $Gl(n, \mathbb{C})/P_j$. Now this homogeneous space is projective, i.e., admits an embedding into a projective space. Explicitly, let $\Lambda\mathbb{C}^n$ be the exterior algebra generated by the basis elements e_a of $\mathbb{C}^n$ and $\Lambda^j\mathbb{C}^n$ the degree j part, i.e., the linear span of elementary wedges $e_{i_1} \wedge e_{i_2} \wedge \cdots \wedge e_{i_j}$. For $W \in Gr_j(\mathbb{C}^n)$ with basis $w_1, w_2, \dots, w_j$ we have the element $w_1 \wedge w_2 \wedge \cdots \wedge w_j$ which is up to multiplication by a non zero scalar independent of the choice of basis. This then defines an embedding $\phi_j : Gr_j(\mathbb{C}^n) \to \mathbb{P}\Lambda^j\mathbb{C}^n$. (If V is a vector space $\mathbb{P}V$ denotes the associated projective space.) The image of ϕ_j is the projectivization of the $Gl(n, \mathbb{C})$ orbit of the highest weight vector $e_1 \wedge e_2 \wedge \cdots \wedge e_j$