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# ON COMPLETE METRICS OF NONNEGATIVE CURVATURE ON 2-PLANE BUNDLES

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**This paper is an attempt to understand the 2-plane bundle case for the converse of the soul theorem due to J. Cheeger and D Gromoll. It is shown that there is a class of 2-plane bundles over certain $S^2$-bundles that carry complete metrics of nonnegative sectional curvature. In particular, every 2-plane bundle and every $S^1$-bundle over the connected sum $CP^n \# \overline{CP}^n$ of $CP^n$ with a negative $CP^n$ carries a 2-parameter family of complete metrics with nonnegative sectional curvature.**

A complete noncompact Riemannian manifold with nonnegative curvature ($K \geq 0$) is diffeomorphic to the normal bundle of a closed totally geodesic submanifold of the noncompact manifold according to a fundamental theorem due to J. Cheeger and D. Gromoll [**4**]. They also proposed the following problem: Which vector bundles over closed manifolds with $K \geq 0$ carry complete metrics with $K \geq 0$?

This paper is an attempt to understand the 2-plane bundle case. For a class of 2-plane bundles and $S^1$-bundles, we show that there exist families of complete metrics with $K \geq 0$. Our main result is the following

**Theorem.** *Every 2-plane bundle and every $S^1$-bundle over the connected sum $\mathbf{C}P^n \# \overline{\mathbf{C}P}^n$ of $\mathbf{C}P^n$ with a negative $\mathbf{C}P^n$ carries a 2-parameter family of complete metrics with $K \geq 0$. More generally, let $(B, h)$ be a Hodge manifold with positive Kahlerian curvature and let $M$ be an associated $S^2$-bundle over $B$. Then, there is a two dimensional subspace $\widetilde{H}^2(M)$, generated by two integral cohomology classes, in the cohomology group $H^*(M)$ such that every principal $S^1$-bundle and every 2-plane bundle over $M$ supported by an Euler class in $\widetilde{H}^2(M)$ carries a 2-parameter family of complete metrics with $K \geq 0$.*

Notice that $B$ can be the product of any number of copies of complex projective spaces with the scaled Fubini-Study metric. When $B = \mathbf{C}P^n$, then $M = \mathbf{C}P^{n+1} \# \overline{\mathbf{C}P}^{n+1}$.

It is worthwhile to point out that examples of complete manifolds with $K \geq 0$ remain scarce. Indeed, all previously known examples of manifolds