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MATCHING THEOREMS FOR TWISTED ORBITAL INTEGRALS

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Let F be a ρ-adic field and E a cyclic extension of F of degree d corresponding to the character κ of $F^{\times}$. For any positive integer m, we consider $H = GL(m, E)$ as a subgroup of $G = GL(md, F)$. In this paper we discuss matching of orbital integrals between H and G. Specifically, ordinary orbital integrals corresponding to regular semisimple elements of H are matched with orbital integrals on G which are twisted by the character κ. For the general situation we only match functions which are smooth and compactly supported on the regular set. If the extension E/F is unramified, we are able to match arbitrary smooth, compactly supported functions.

§1. Introduction.

Let F be a locally compact, non-discrete, nonarchimedean local field of characteristic zero. Let κ be a unitary character of $F^{\times}$ of order d, and let E be the cyclic extension of F corresponding to κ. Let m and n be positive integers with $n = md$ and write $G = GL(n, F)$, $H = GL(m, E)$. H can be identified with a subgroup of G. In this paper we discuss matching of orbital integrals between H and G. Specifically, ordinary orbital integrals corresponding to regular semisimple elements of H are matched with orbital integrals on G which are twisted by the character κ. For the general situation we only match functions which are smooth and compactly supported on the regular set. If the extension E/F is unramified, we are able to match arbitrary smooth, compactly supported functions.

Extend κ to a character of G by $\kappa(g) = \kappa(\det g)$ and let

$$G_0 = \{g \in G : \kappa(g) = 1\}.$$

G_0 is an open normal subgroup of G of finite index and $H \subset G_0$. Let $C_c^{\infty}(G)$ denote the set of locally constant, compactly supported, complex-valued functions on G. For any $\gamma \in G$ we let G_γ denote the centralizer of $\gamma \in G$. If $G_\gamma \subset G_0$, let

$$\Lambda_\kappa^G(f, \gamma) = \int_{G_\gamma \backslash G} f(x^{-1}\gamma x)\kappa(x)dx, f \in C_c^{\infty}(G),$$