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ON MODULI OF INSTANTON BUNDLES ON $\mathbb{P}^{2n+1}$

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Let $\mathrm{MI}_{\mathbb{P}^{2n+1}}(k)$ be the moduli space of stable instanton bundles on $\mathbb{P}^{2n+1}$ with $c_2 = k$. We prove that $\mathrm{MI}_{\mathbb{P}^{2n+1}}(2)$ is smooth, irreducible, unirational and has zero Euler-Poincaré characteristic, as it happens for $\mathbb{P}^3$. We find instead that $\mathrm{MI}_{\mathbb{P}^5}(3)$ and $\mathrm{MI}_{\mathbb{P}^5}(4)$ are singular.

1. Definition and preliminaries.

Instanton bundles on a projective space $\mathbb{P}^{2n+1}(\mathbb{C})$ were introduced in [**OS**] and [**ST**]. In [**AO**] we studied their stability, proving in particular that special symplectic instanton bundles on $\mathbb{P}^{2n+1}$ are stable, and that on $\mathbb{P}^5$ every instanton bundle is stable.

In this paper we study some moduli spaces $\mathrm{MI}_{\mathbb{P}^{2n+1}}(k)$ of stable instanton bundles on $\mathbb{P}^{2n+1}$ with $c_2 = k$. For $k = 2$ we prove that $\mathrm{MI}_{\mathbb{P}^{2n+1}}(2)$ is *smooth, irreducible, unirational and has zero Euler-Poincaré characteristic* (Theor. 3.2), just as in the case of $\mathbb{P}^3$ [**Har**].

We find instead that $\mathrm{MI}_{\mathbb{P}^5}(k)$ is *singular* for $k = 3, 4$ (theor. 3.3), which is not analogous with the case of $\mathbb{P}^3$ [**ES**], [**P**]. To be more precise, *all points corresponding to symplectic instanton bundles are singular.* Theor. 3.3 gives, to the best of our knowledge, the first example of a singular moduli space of stable bundles on a projective space. The proof of Theorem 3.3 needs help from a personal computer in order to calculate the dimensions of some cohomology group [**BaS**].

We recall from [**OS**], [**ST**] and [**AO**] the definition of instanton bundle on $\mathbb{P}^{2n+1}(\mathbb{C})$.

Definition 1.1. A vector bundle E of rank $2n$ on $\mathbb{P}^{2n+1}$ is called an instanton bundle of quantum number k if

(i) The Chern polynomial is $c_t(E) = (1-t^2)^{-k} = 1 + kt^2 + \binom{k+1}{2}t^2 + \dots$

(ii) $E(q)$ has natural cohomology in the range $-2n-1 \le q \le 0$ (that is $h^i(E(q)) \neq 0$ for at most one $i = i(q)$)

(iii) $E|_r \simeq \mathcal{O}_r^{2n}$ for a general line r.

Every instanton bundle is simple [**AO**]. There is the following characterization: